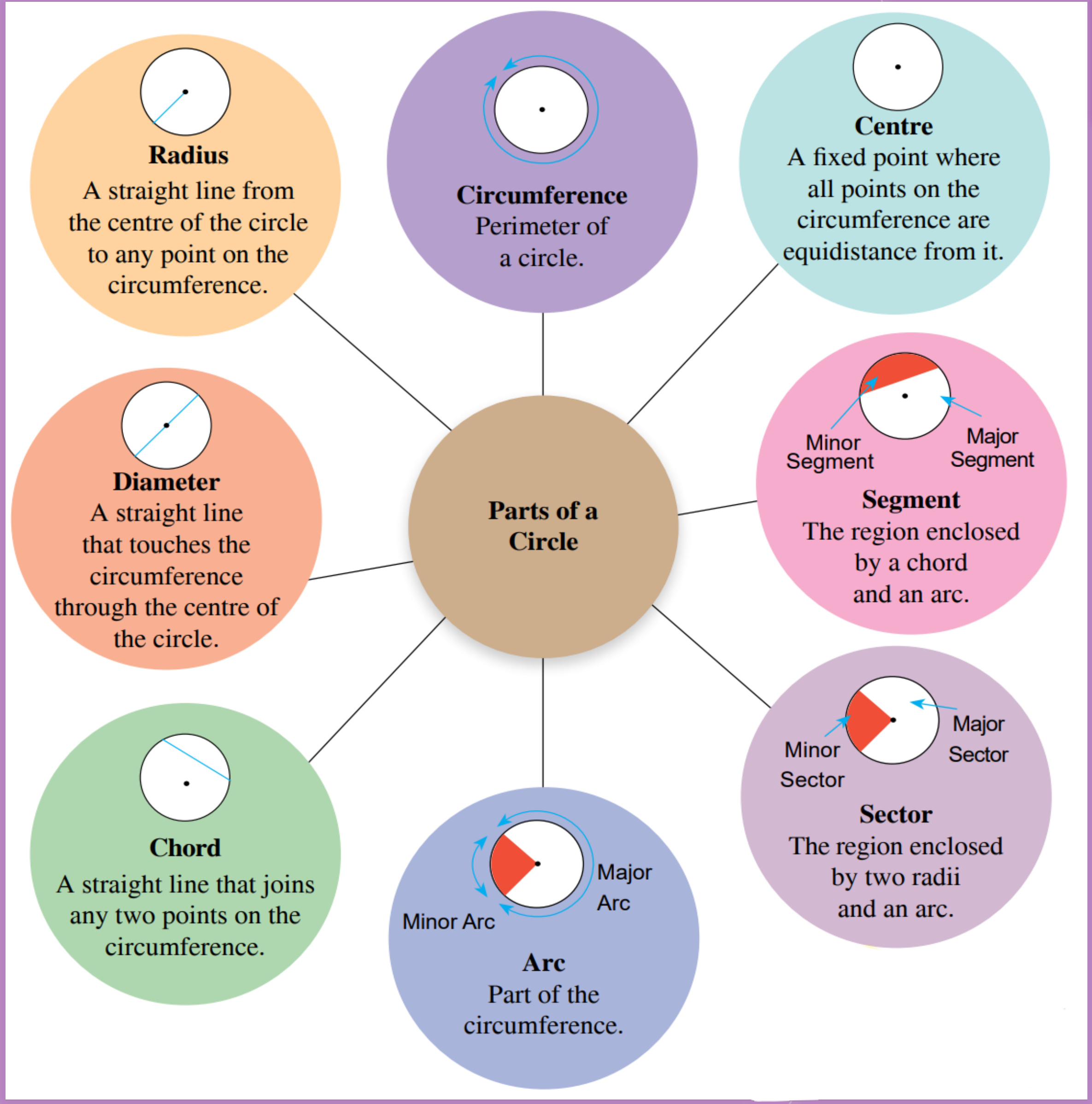
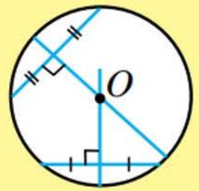
Chapter 5 Form 2

Circles



Perpendicular bisectors of two chords meet at the centre of the circle.

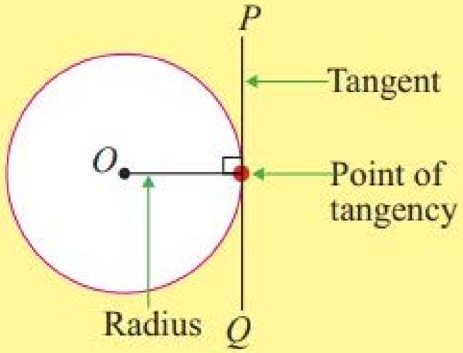


$$\text{Circumference} = \pi \times 2 \times \text{radius} = 2\pi r$$

$$\text{area of circle} = \pi r^2$$

$$\frac{\text{Length of arc}}{2\pi r} = \frac{\theta}{360^\circ}$$

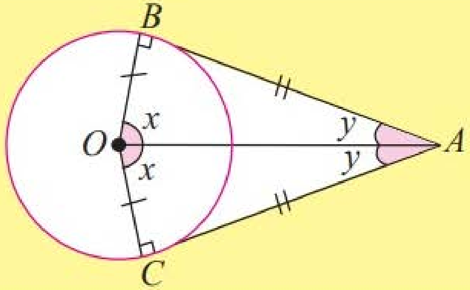
$$\frac{\text{Area of sector}}{\pi r^2} = \frac{\theta}{360^\circ}$$



The radius of a circle that intersects with tangent to the circle at the point of tangency will form a 90° angle with the tangent.

Chapter 6 Form 3

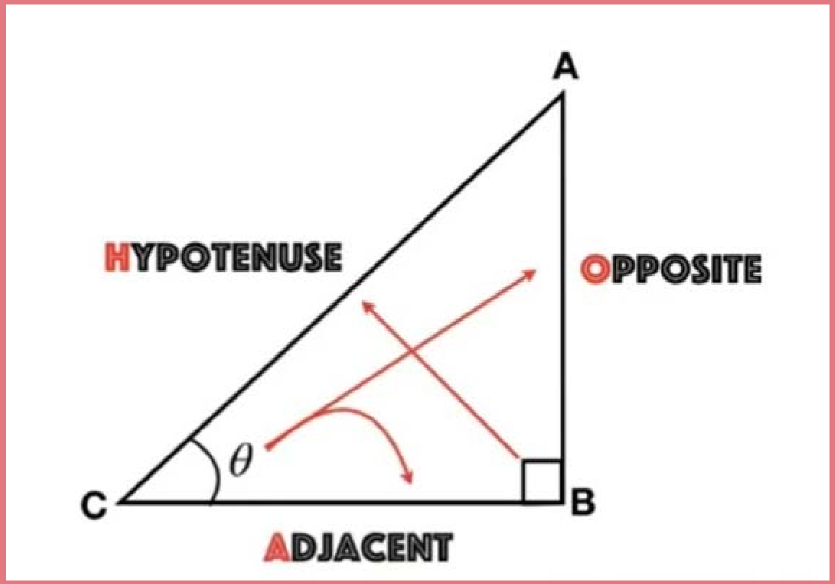
Angles and Tangents of Circles



If two tangents to a circle with centre O and points of tangency B and C meet at point A, then

- $BA = CA$
- $\angle BOA = \angle COA$
- $\angle OAB = \angle OAC$

Chapter 5 Form 3
Trigonometric Ratio



$$\text{sine} = \frac{\text{opposite side}}{\text{hypotenuse}} \quad \textbf{SOH}$$

$$\text{cosine} = \frac{\text{adjacent side}}{\text{hypotenuse}} \quad \textbf{CAH}$$

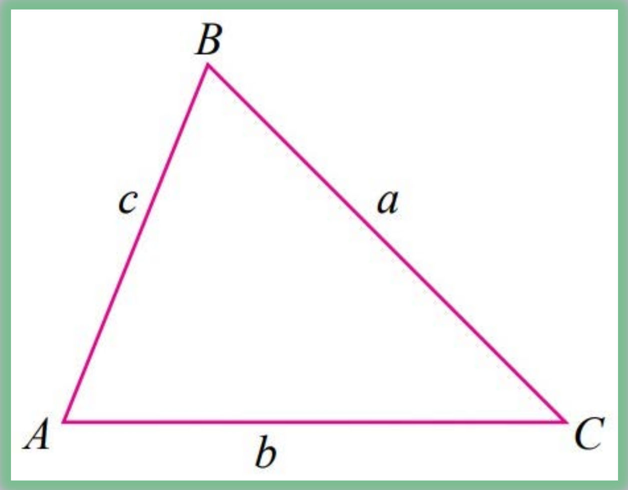
$$\text{tangent} = \frac{\text{opposite side}}{\text{adjacent side}} \quad \textbf{TOA}$$

Additional info:

Pythagoras Theorem

$$AC^2 = AB^2 + BC^2$$

Chapter 9 Form 4
Solution of Triangles



Sine Rule:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Cosine Rule:

$$a^2 = b^2 + c^2 - 2bc \cos A$$

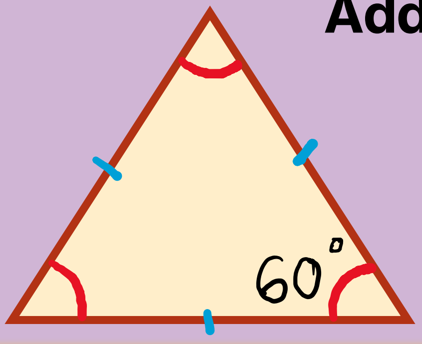
Area of triangle:

$$A = \frac{1}{2} ab \sin C$$

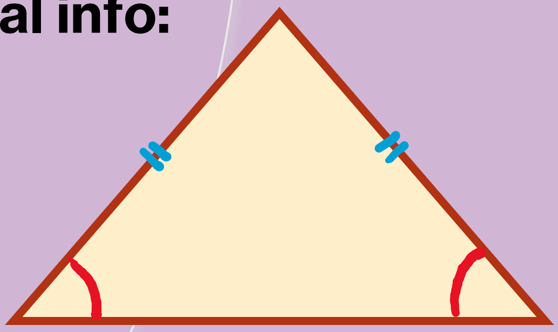
OR

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

Additional info:

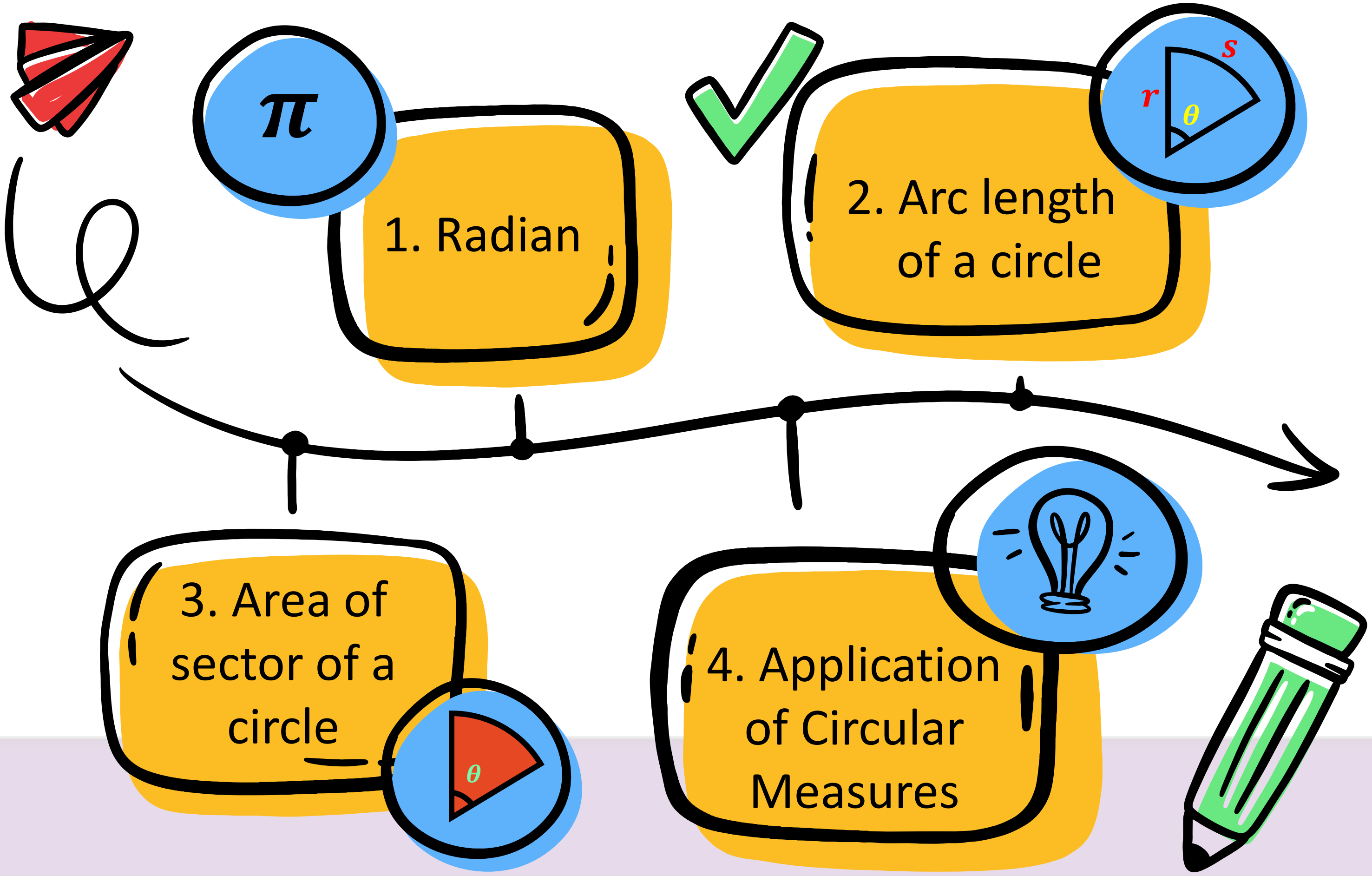


Equilateral triangle



Isosceles triangle

Circular Measure



1.1 Radian

One radian

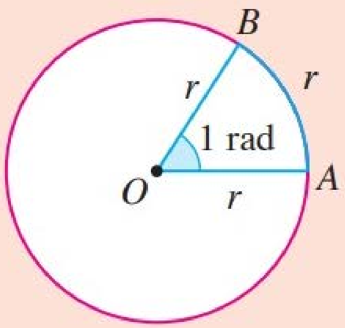
$2\pi\ rad = 360°$

$\pi\ rad = 180°$

$1\ rad = \frac{180°}{\pi}$

$1° = \frac{\pi}{180°}$

One radian is the measure of an angle subtended at the centre of a circle by an arc whose length is the same as the radius of the circle.



Degree $\times \frac{\pi}{180°}$ → Radian

Radian $\times \frac{180°}{\pi}$ → Degree

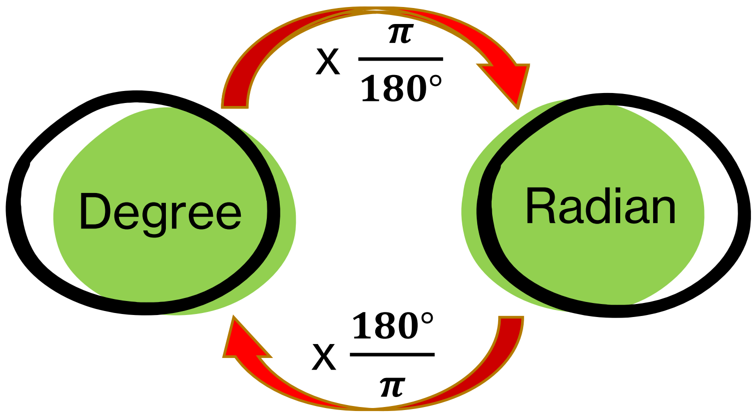
Arc AB = r, then $\angle BOA$ = 1 rad.
Arc AB = 2r, then $\angle BOA$ = 2 rad.
Arc AB = πr, then $\angle BOA$ = π rad.
Arc AB = 2πr, then $\angle BOA$ = 2π rad.

Example 1

Convert each of the following angles into degrees.
[Use $\pi = 3.142$]

(a) $\frac{3}{4}\pi$ rad

(b) 1.04 rad



Solution

$$\boldsymbol{\pi} \textbf{ rad} = \mathbf{180°}$$

$$\textbf{(a)}\ \frac{3}{4}\pi \text{ rad} = \frac{3}{4}\pi \times \frac{180°}{\pi}$$

$$= \frac{3}{4} \times 180°$$

$$= 135°$$

$$\textbf{(b)}\ 1.04 \text{ rad} = 1.04 \times \frac{180°}{\pi}$$

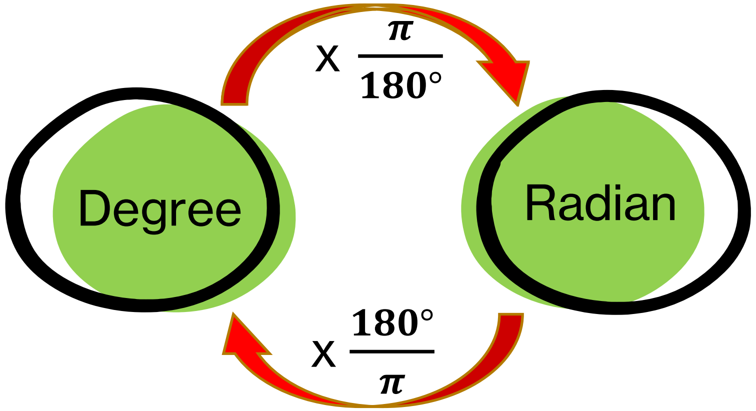
$$= 1.04 \times \frac{180°}{3.142}$$

$$= 59.58° \quad @ \quad 59°\,35'$$

Example 2

(a) Convert angles $140°$ into radians, in terms of π.

(b) Convert $225°$ and $32°\,15'$ into radians.
[Use $\pi = 3.142$]



Solution

$180° = \pi$ rad

(a) $140° = 140° \times \dfrac{\pi}{180°}$

$= \dfrac{7}{9}\pi$ rad

(b) $225° = 225° \times \dfrac{\pi}{180°}$

$= 225° \times \dfrac{3.142}{180°}$

$= 3.928$ rad

$32°\,15' = 32°\,15' \times \dfrac{\pi}{180°}$

$= 32°\,15' \times \dfrac{3.142}{180°}$

$= 0.5629$ rad

Tips

* Follow what the questions want.

1.2 Arc Length of a Circle

Form 2

$$\frac{\text{Minor arc length } AB}{\text{Circumference}} = \frac{\theta}{360^\circ}$$

Angle in degrees

$$\frac{\text{Minor arc length } AB}{\text{Circumference}} = \frac{\theta}{2\pi}$$

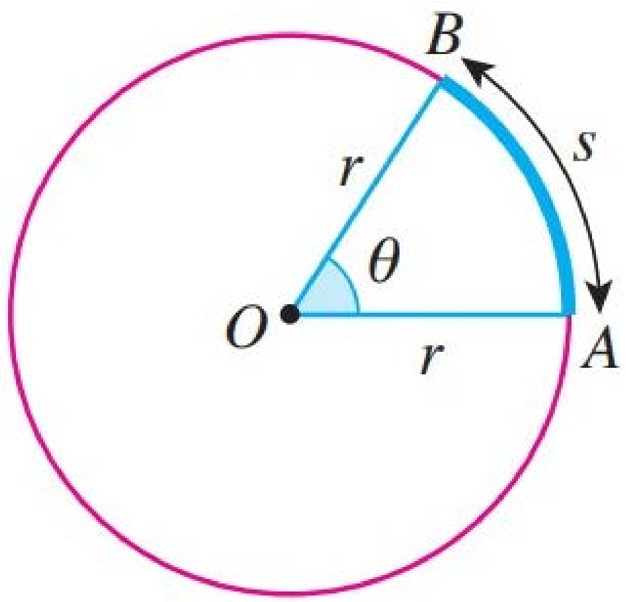
Angle in radians

$$\frac{s}{2\pi r} = \frac{\theta}{2\pi}$$

$$s = \frac{\theta}{2\pi} \times 2\pi r$$

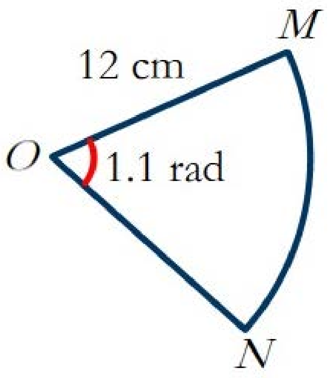
$$\boldsymbol{s = r\theta}$$

$\boldsymbol{\theta}$ must be in radians



Example 3

(a) Find the arc length *MN*, in cm, of the sector *MON* with the centre *O*.



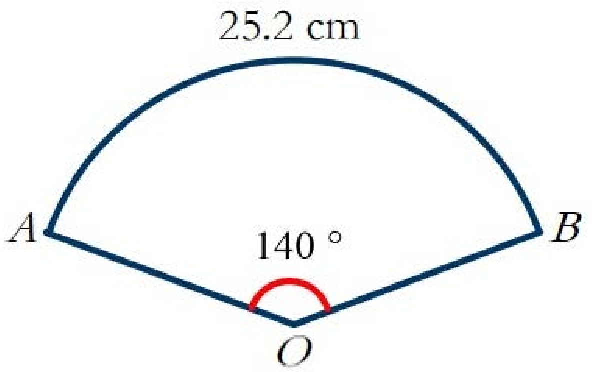
Solution

$$s = r\theta$$

$$\text{Arc } MN = 12 \times 1.1$$

$$= 13.20 \text{ cm}$$

(b) Find the radius, in cm, of the sector *AOB* with the centre *O*. [Use $\pi = 3.142$]



Solution

$$s = r\theta$$

θ must be in radian

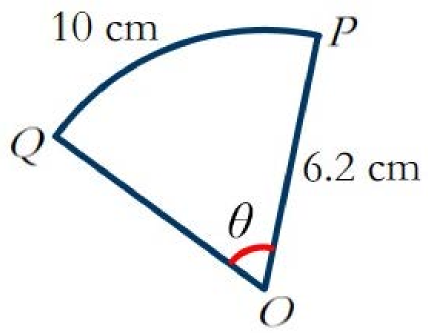
$$25.2 = r \times \left(140° \times \frac{3 \cdot 142}{180°}\right)$$

$$25.2 = r \times 2.4438$$

Round off to at least 4 d.p

$$r = 10.31 \text{ cm}$$

(c) Diagram below shows the sector of *POQ* with the centre *O*. Find the value of θ, in radian.



Solution

$$s = r\theta$$

$$10 = 6.2 \times \theta$$

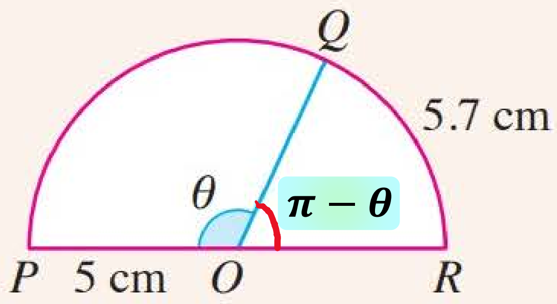
$$\theta = \frac{10}{6.2}$$

$$\theta = 1.613 \text{ rad}$$

Example 4

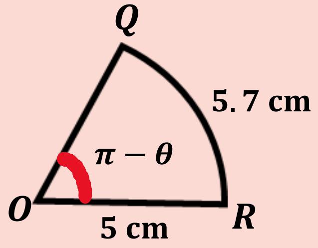
π rad = 180°

The diagram on the right shows semicircle *OPQR* with a radius of 5 cm. Given that the arc length *QR* is 5.7 cm, calculate
(a) the value of θ, in radians,
(b) the arc length *PQ*, in cm.
[Use $\pi = 3.142$]



Solution

(a) $\angle QOR = (\pi - \theta)$ rad



$s = r\theta$

$5.7 = 5 \times (\pi - \theta)$

$5.7 = 5 \times (3.142 - \theta)$

$\frac{5.7}{5} = 3.142 - \theta$

$\theta = 2.002$ rad

Alternative Method:

Arc length *PQ*
$= \pi r - 5.7$
$= 3.142(5) - 5.7$
$= 10.01$ cm

$s = r\theta$

$10.01 = 5\theta$

$\theta = 2.002$ rad

$s = r\theta$

(b) Arc length $PQ = 5\,(2.002)$

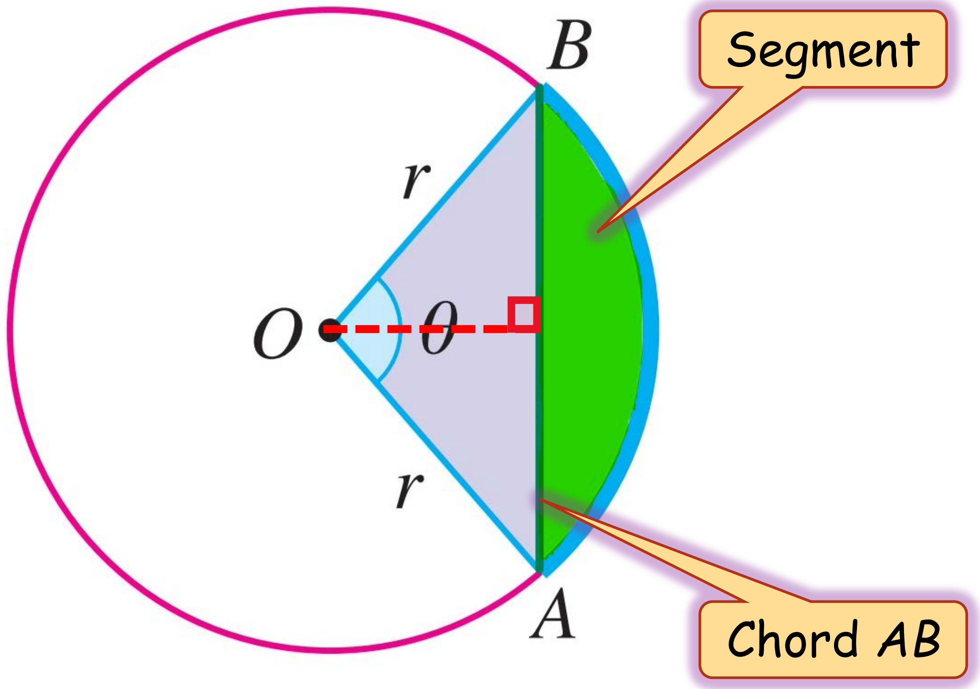
$= 10.01$ cm

Alternative Method:

Arc length *PQ*
$= \pi r - 5.7$
$= 3.142(5) - 5.7$
$= 10.01$ cm

1.2 Arc Length of a Circle

=> **Determining the perimeter of segment of a circle**



Perimeter of segment
=Arc length *AB* + Chord *AB*

$s = r\theta$

Cosine Rule:

$a^2 = b^2 + c^2 - 2bc\cos A$

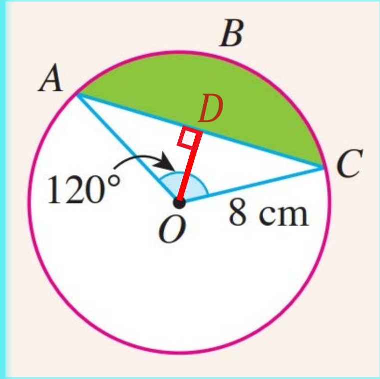
OR

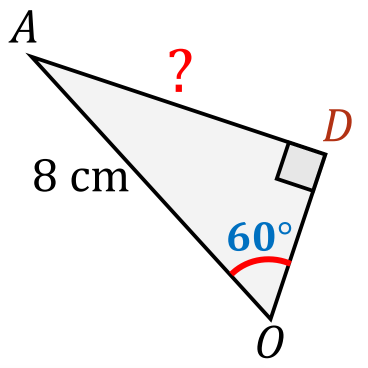
Trigonometric Ratio:

SOH *CAH* *TOA*

Example 5

The diagram on the right shows a circle with centre O and radius of 8 cm. The chord AC subtends an angle of 120° at the centre of the circle. Calculate the perimeter of the shaded segment ABC.
[Use $\pi = 3.142$]





Solution

Cosine rule:
$a^2 = b^2 + c^2 - 2bc\cos A$

Perimeter of shaded region
$= Arc\ ABC + Chord\ AC$

$s = r\theta$

$$s_{ABC} = 8 \times \left(120° \times \frac{3 \cdot 142}{180°}\right)$$
$$= 16.76 \text{ cm}$$

$$AC^2 = 8^2 + 8^2 - 2(8)(8)(cos120°)$$
$$= 192$$
$$AC = \sqrt{192}$$
$$= 13.86 \text{ cm}$$

$$Perimeter\ of\ shaded\ region = 16.76 + 13.86$$
$$= 30.62 \text{ cm}$$

Alternative Method (find AC):

$$sin\angle AOD = \frac{AD}{OA}$$

$$AD = OA\ sin\angle AOD$$
$$AD = 8\ sin60°$$
$$= 6.928 \text{ cm}$$

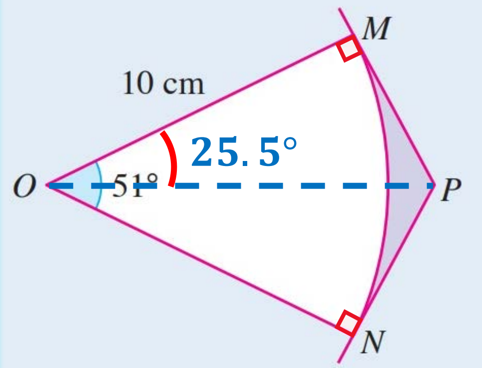
Thus, $AC = 2AD$
$$AC = 2(6.928)$$
$$= 13.86 \text{ cm}$$

Example 6

The diagram on the right shows a part of a circle with centre O and a radius of 10 cm. Tangent to the circle at point M point N meet at P and $\angle MON = 51°$. Calculate

(a) Arc length MN, in cm,

(b) the perimeter of the shaded region, in cm.



Solution

(a) $s = r\theta$ — θ in radians

$$Arc\ MN = 10 \times \left(51° \times \frac{3 \cdot 142}{180°}\right)$$

$$= 10 \times 0.8902$$

Round off to at least 4 d.p

$$= 8.902 \text{ cm}$$

// 8.901 cm

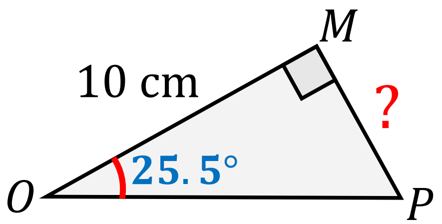
(b) $Perimeter\ of\ shaded\ region = Arc\ MN + MP + PN$

$$\tan\angle MOP = \frac{MP}{OM}$$

$$MP = OM \tan\angle MOP$$

$$MP = 10 \tan 25.5°$$

$$= 4.7698 \text{ cm}$$



$$Perimeter\ of\ shaded\ region = 8.902 + 2(4.7698)$$

$$= 18.44 \text{ cm}$$

1.3 Area of a sector of a Circle

Form 2

$$\frac{\text{Area of minor sector } AOB}{\text{Area of } circle} = \frac{\theta}{360°}$$

Angle in degrees

$$\frac{\text{Area of minor sector } AOB}{\text{Area of } circle} = \frac{\theta}{2\pi}$$

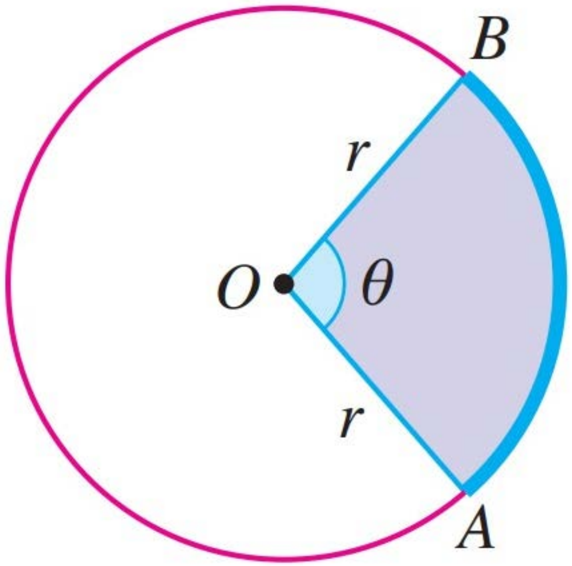
Angle in radians

$$\frac{A}{\pi r^2} = \frac{\theta}{2\pi}$$

$$A = \frac{\theta}{2\pi} \times \pi r^2$$

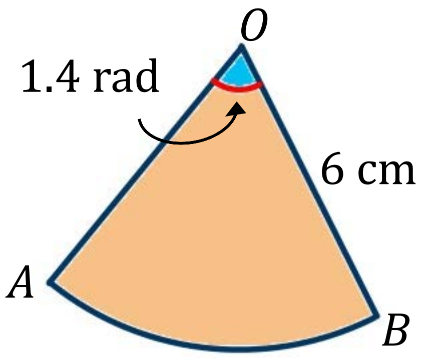
$$\boldsymbol{A = \frac{1}{2} r^2 \theta}$$

$\boldsymbol{\theta}$ must be in radians



Example 7

(a) Diagram below shows a sector AOB with the centre O. Determine the area, in cm^2.



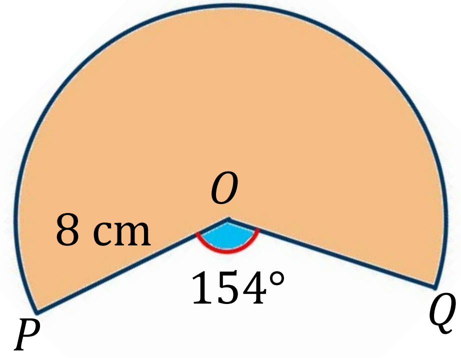
Solution

$$\boldsymbol{A = \frac{1}{2}r^2\theta}$$

$$A = \frac{1}{2}(6)^2(1.4)$$

$$= 25.20 \text{ cm}^2$$

(b) The diagram on the right shows a major sector POQ with the centre O. Find the area, in cm^2.
[Use $\pi = 3.142$]



Solution

Reflex angle POQ in radians

$$= (360° - 154°) \times \frac{3 \cdot 142}{180°}$$

$$= 206° \times \frac{3 \cdot 142}{180°}$$

$$= 3.5958 \text{ rad}$$

$$\boldsymbol{A = \frac{1}{2}r^2\theta}$$

θ **must be in radian**

$$A = \frac{1}{2}(8)^2(3.5958)$$

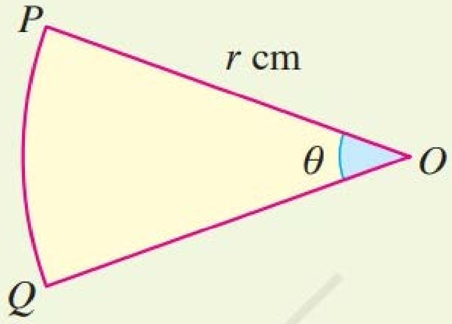
$$= 115.07 \text{ cm}^2$$

Example 8

The diagram on the right shows a sector POQ which subtends an angle of θ radians and has a radius of r cm. Given that the area of the sector POQ is 35 cm², find

(a) the value of r if $\theta = 0.7$ rad,

(b) the value of θ if the radius is 11 cm.



Solution

(a)

$$A = \frac{1}{2}r^2\theta$$

$$\text{Area sector } POQ = 35 \text{ cm}^2$$

$$\frac{1}{2}r^2\theta = 35$$

$$\frac{1}{2}r^2(0.7) = 35$$

$$r^2 = \frac{35 \times 2}{0.7}$$

$$r^2 = 100$$

$$r = \sqrt{100}$$

$$r = 10 \text{ cm}$$

(b)

$$\text{Area sector } POQ = 35 \text{ cm}^2$$

$$\frac{1}{2}r^2\theta = 35$$

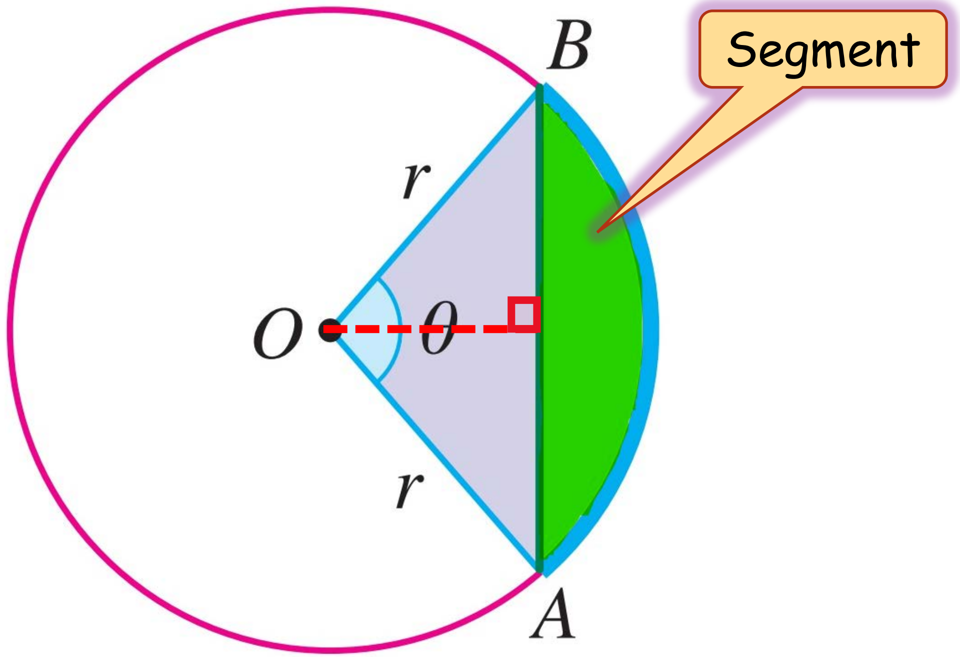
$$\frac{1}{2}(11)^2\theta = 35$$

$$\theta = \frac{35 \times 2}{(11)^2}$$

$$\theta = 0.5785 \text{ rad}$$

1.3 Area of a sector of a Circle

=> Determining the area of segment of a circle



Area of segment
=Area sector *AOB* – Area triangle *AOB*

$$A = \frac{1}{2}r^2\theta$$

$$A = \frac{1}{2}ab\sin C$$

OR

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

OR

$$A = \frac{1}{2} \times base \times height$$

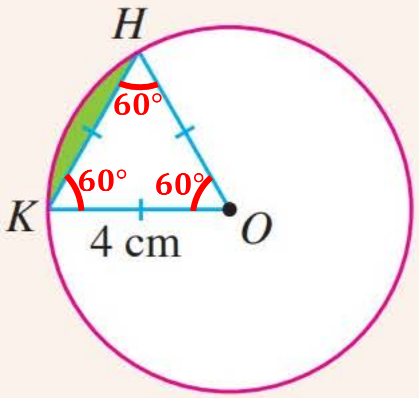
Trigonometric Ratio / Pythagoras Theorem

Example 9

The diagram on the right shows sector HOK of a circle with centre O and a radius of 4 cm. The length of chord HK is the same as the length of the radius of the circle. Calculate

(a) $\angle HOK$, in radians,

(b) the area of the shaded segment, in cm^2.



$s = r\theta$

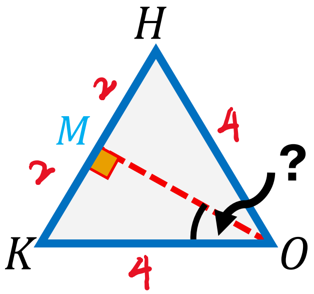
$A = \frac{1}{2}r^2\theta$

Solution 9 (a)

(a) $r = \text{chord } HK = 4 \text{ cm}$

Since $OK = OH = \text{chord } HK$

Thus, triangle *OKH* is an equilateral triangle.



$$\angle HOK = \frac{180°}{3} \times \frac{\pi}{180°}$$
$$= 60° \times \frac{3 \cdot 142}{180°}$$
$$= 1.047 \text{ rad}$$

OR

Alternative Method:

$$\sin\angle KOM = \frac{2}{4}$$
$$\angle KOM = 30°$$
$$\angle HOK = 2(30°) \times \frac{3 \cdot 142}{180°}$$
$$= 1.047 \text{ rad}$$

$\sin^{-1}\left(\frac{2}{4}\right)$

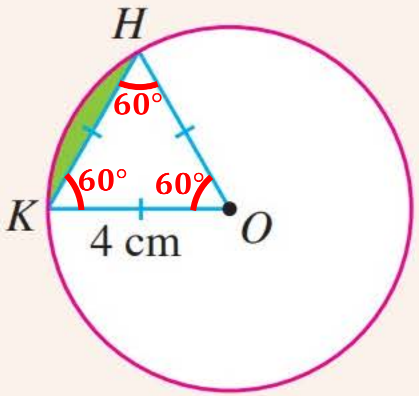
Tips to find angle:

- Look at this formula $s = r\theta$ and $A = \frac{1}{2}r^2\theta$

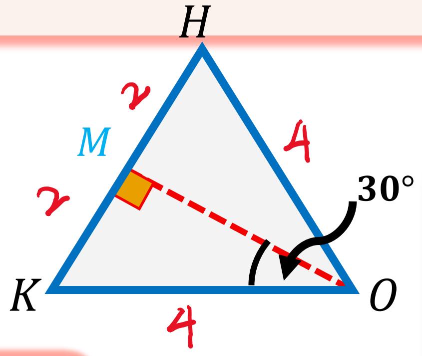
*If both cannot be used, then try use the **triangle (right-angled/equilateral/isosceles)**.

Example 9

The diagram on the right shows sector *HOK* of a circle with centre *O* and a radius of 4 cm. The length of chord *HK* is the same as the length of the radius of the circle. Calculate
(a) $\angle HOK$, in radians,
(b) the area of the shaded segment, in cm².



Solution 9 (b)



(*b*) **Area of shaded segment**
= Area of sector *HOK* − Area of triangle *HOK*

$$A = \frac{1}{2}r^2\theta$$

$$Asect = \frac{1}{2}(4)^2(1.047)$$
$$= 8.376 \text{ cm}^2$$

$$A = \frac{1}{2}ab\sin C$$

$$A\ \Delta HOK = \frac{1}{2}(4)(4)(\sin 60°)$$
$$= 6.928 \text{ cm}^2$$

Area of shaded segment $= 8.376 - 6.928$
$$= 1.448 \text{ cm}^2$$

$1.4478 \leftrightarrow 1.4504$

OR

Alternative Method:

$$A = \frac{1}{2} \times base \times height$$

$$OM = \sqrt{4^2 - 2^2} = \sqrt{12}$$

$$Area\ \Delta HOK = \frac{1}{2} \times HK \times OM$$
$$= \frac{1}{2} \times 4 \times \sqrt{12}$$
$$= 6.928 \text{ cm}^2$$

OR

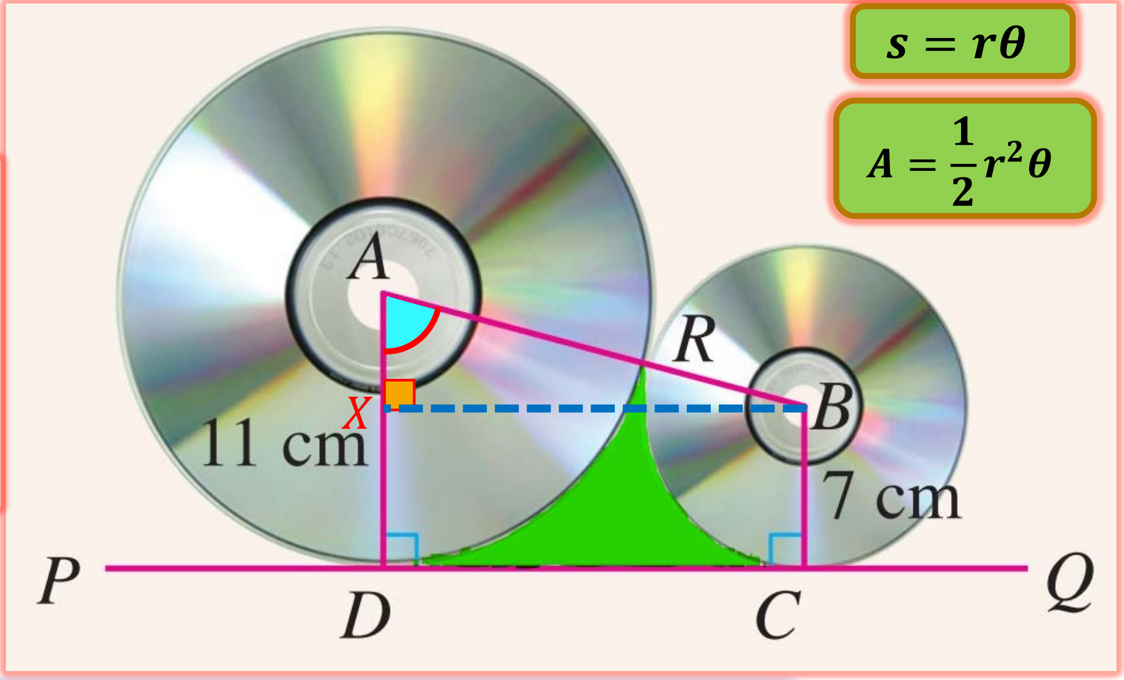
$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

$$s = \frac{4+4+4}{2} = 6$$

$$A = \sqrt{6(6-4)(6-4)(6-4)}$$
$$= 6.928$$

Example 10

The diagram on the right shows two discs with radii 11 cm and 7 cm touching each other at R. The discs are on a straight line $PDCQ$.
(a) Calculate $\angle BAD$, in degrees.
(b) Subsequently, find the shaded area in cm^2.

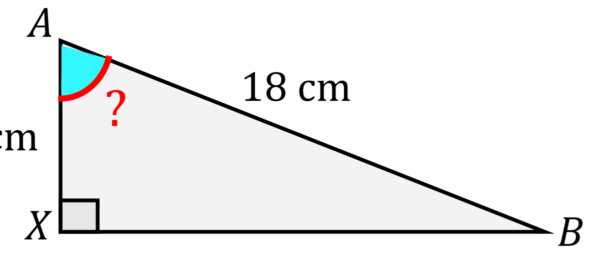


$s = r\theta$

$A = \frac{1}{2}r^2\theta$

Solution

(a)



$$\cos\angle BAX = \frac{AX}{AB}$$

$$\cos\angle BAD = \frac{4}{18}$$

$\cos^{-1}\left(\frac{4}{18}\right)$

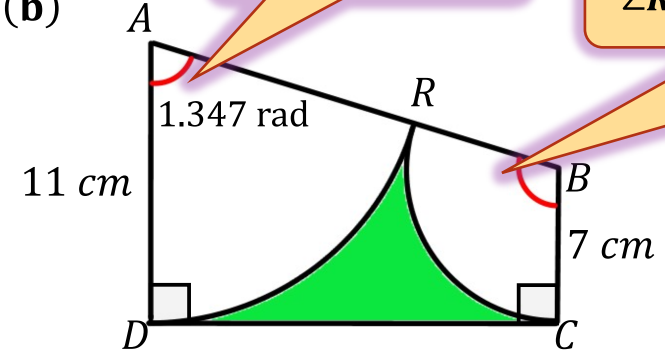
$$\angle BAD = 77.16°$$

(b)

$180° = \pi$ rad

$77.16° \times \frac{3 \cdot 142}{180°}$

$\angle RBC = (180° - 77.16°) \times \frac{3 \cdot 142}{180°} = 1.795$ **rad**



$$CD = \sqrt{18^2 - 4^2} = 17.55 \text{ cm}$$

$$\text{Area Trapezium } ABCD = \frac{1}{2} \times (11 + 7) \times 17.55$$

$$= 157.95 \text{ cm}$$

$$\text{Area Sector } ADR = \frac{1}{2}(11)^2(1.347)$$

$$= 81.49 \text{ cm}^2$$

$$\text{Area Sector } BCR = \frac{1}{2}(7)^2(1.795)$$

$$= 43.98 \text{ cm}^2$$

Area of shaded region

$= 157.95 - 81.49 - 43.98$

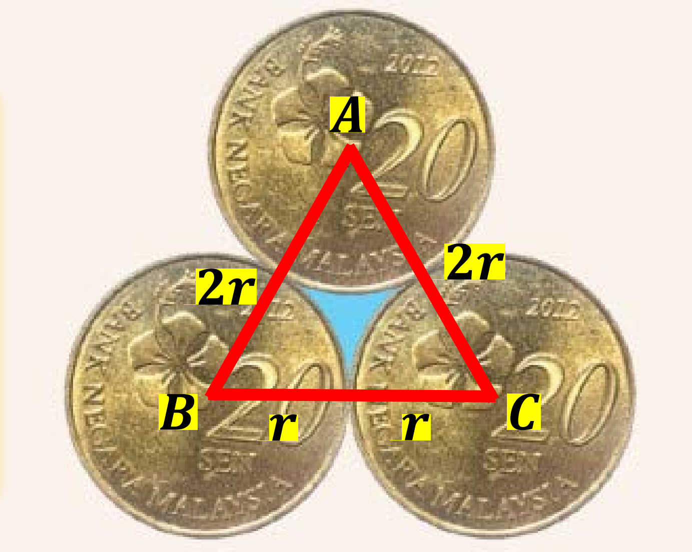
$= 32.48 \text{ cm}^2$

$32.48 \leftrightarrow 32.51$

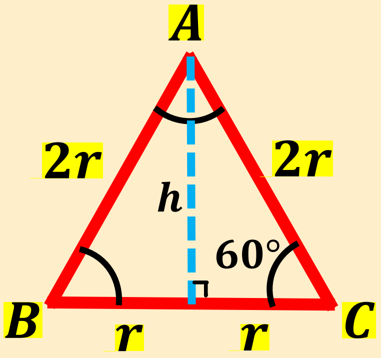
1.4 Application of Circular Measures

Example 11

In the diagram on the right are three identical 20 cent coins with the same radii and touching each other. If the blue coloured region has an area of 12.842 mm², find the radius of each coin, in mm.



Solution



$$h = \sqrt{(2r)^2 - r^2}$$
$$= \sqrt{4r^2 - r^2}$$
$$= \sqrt{3r^2}$$
$$= 1.732\,r$$

OR

$$\sin 60° = \frac{h}{2r}$$
$$h = 1.732\,r$$

Area ΔABC − Area of 3 sectors = Area coloured region

$$\frac{1}{2} \times b \times h - 3\left(\frac{1}{2} r^2 \theta\right) = 12.842$$

$$\frac{1}{2} \times 2r \times 1.732\,r - 3\left(\frac{1}{2} r^2 \left(60° \times \frac{3 \cdot 142}{180°}\right)\right) = 12.842$$

OR

$$A = \frac{1}{2} ab \sin C$$
$$A\ \Delta ABC = \frac{1}{2}(2r)(2r)\ (\sin 60°)$$
$$= 1.732\,r^2$$

$$1.732\,r^2 - 1.571 r^2 = 12.842$$
$$0.161\,r^2 = 12.842$$
$$r^2 = 79.764$$
$$r = \sqrt{79.764}$$
$$r = 8.931 \text{ mm}$$

Past Year Papers...

Are you
ready?

Example 12

Diagram 14 shows an isosceles triangle OPQ, such that $OP = PQ = 25$ cm and $\angle POQ = 0.87$ radians. PR is an arc of a circle with centre O.

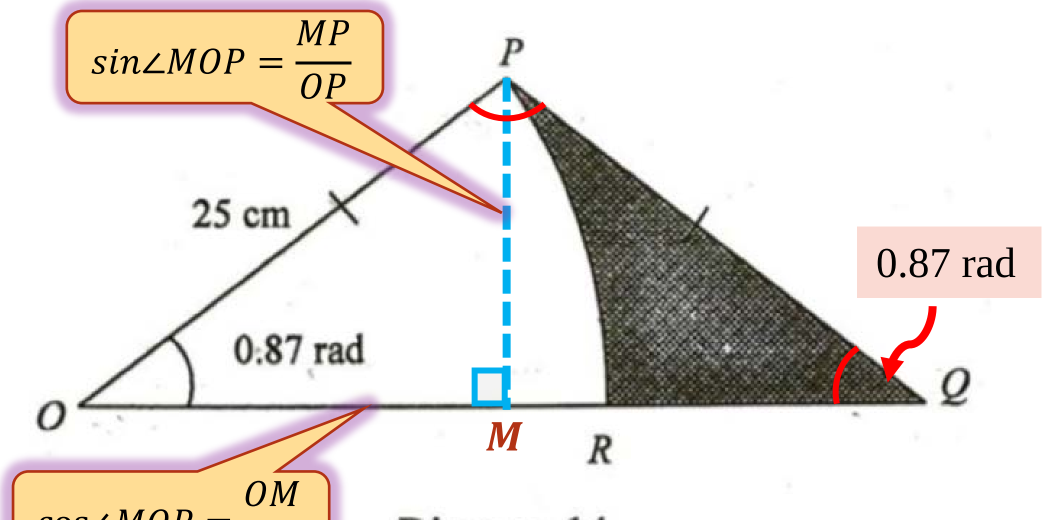


Diagram 14

Find the area, in cm^2, of the shaded region, correct to two decimal places.

[3 *marks*]

[Use $\pi = 3.142$]

Solution

Area shaded region = Area ΔOPQ − Area sector OPR

Area Sector $OPR = \frac{1}{2}(25)^2(0.87)$ ✓ $A = \frac{1}{2}r^2\theta$

$= 271.875 \text{ cm}^2$

$0.87 \text{ rad} = 0.87 \times \frac{180°}{3.142}$

$= \mathbf{49.84°}$

$\angle OPQ = 180° - 2(49.84°) = \mathbf{80.32°}$

$A = \frac{1}{2}ab\sin C$

$A\ \Delta OPQ = \frac{1}{2}(25)(25)(\sin 80.32°)$ ✓

$= 308.05 \text{ cm}^2$

OR

Alternative Method:

$A = \frac{1}{2} \times base \times height$

$OM = 25\cos 49.84° = 16.12$ cm

$MP = 25\sin 49.84° = 19.11$ cm

$Area\ \Delta OPQ = \frac{1}{2} \times OQ \times MP$

$= \frac{1}{2} \times 2(16.12) \times 19.11$

$= 308.05 \text{ cm}^2$

Area shaded region $= \frac{1}{2}(25)(25)(\sin 80.32°) - \frac{1}{2}(25)^2(0.87)$ ✓

$= 308.05 - 271.875$

$= 36.18 \text{ cm}^2$ ✓

Example 13

Diagram 5 shows a circle with centre O.

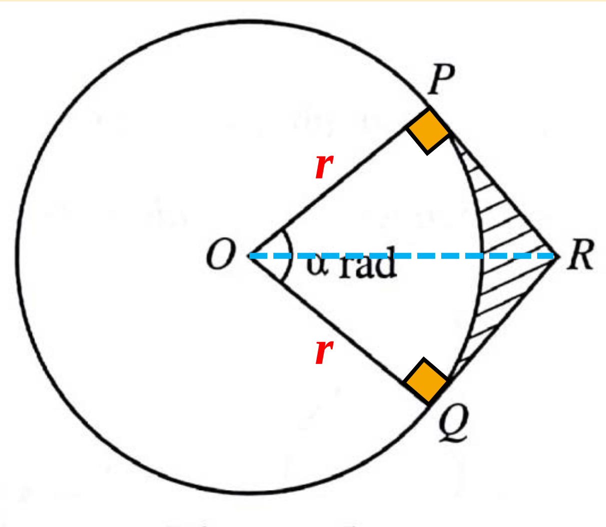


Diagram 5

PR and QR are tangents to the circle at points P and Q respectively. It is given that the length of minor arc PQ is 4 cm and $OR = \frac{5}{\alpha}$ cm.

Express in terms of α

(a) the radius r, of the circle,

(b) the area, A, of the shaded region.

[4 *marks*]

Solution

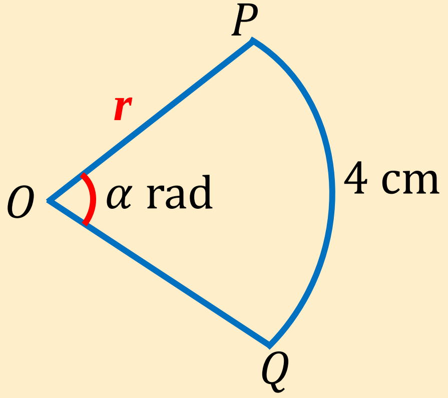
(a)

$$s = 4$$

$s = r\theta$

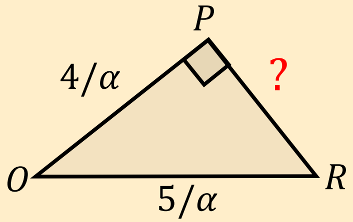
$$r\alpha = 4$$

$$r = \frac{4}{\alpha}$$



(b)

Area shaded region = Area quad. $OPRQ$ − Area sector POQ



$$PR = \sqrt{\left(\frac{5}{\alpha}\right)^2 - \left(\frac{4}{\alpha}\right)^2}$$

$$PR = \sqrt{\frac{25}{\alpha^2} - \frac{16}{\alpha^2}}$$

$$PR = \sqrt{\frac{9}{\alpha^2}} = \frac{3}{\alpha}$$

$$\text{Area S/R} = 2\left(\frac{1}{2} \times b \times h\right) - \frac{1}{2}r^2\theta$$

$$= 2\left(\frac{1}{2} \times \frac{3}{\alpha} \times \frac{4}{\alpha}\right) - \frac{1}{2}\left(\frac{4}{\alpha}\right)^2(\alpha)$$

$$= \frac{12}{\alpha^2} - \frac{8\alpha}{\alpha^2}$$

$$= \frac{12 - 8\alpha}{\alpha^2}$$

Example 15

Diagram 1 shows a sector *POA* with centre *O*.

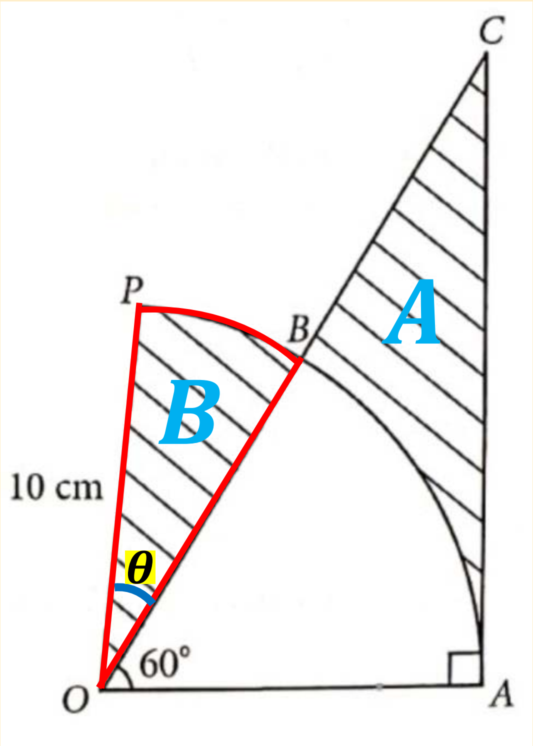


Diagram 1

It is given that the length of arc *PB* is 2.56 cm.
[Use $\pi = 3.142$]

Calculate

(a) $\angle POB$ in radians, [2 *marks*]

(b) the area, in cm^2, of the shaded region. [4 *marks*]

Solution

(a)

$$s = 2.56$$

$s = r\theta$

$$(10)\,\theta = 2.56$$

$$\theta = 0.2560 \text{ rad}$$

(b)

Area shaded region
= Area A + Area B
= (Area ΔOAC − Area sector AOB) + Area sector POB

$$= \left(\frac{1}{2} \times b \times h - \frac{1}{2} r^2 \theta\right) + \frac{1}{2} r^2 \theta$$

$$= \left(\frac{1}{2} \times 10 \times 17.32 - \frac{1}{2}(10)^2(1.047)\right) + \frac{1}{2}(10)^2(0.2560)$$

$$= 34.25 + 12.8$$

$$= 47.05 \text{ cm}^2$$

$\tan 60^\circ = \frac{h}{10}$
$h = 10 \times \tan 60^\circ$
$= 17.32$ cm

$\left(60^\circ \times \frac{3 \cdot 142}{180^\circ}\right)$
$= \mathbf{1.047}$ **rad**

Example 16

Diagram 3 shows the area for throwing javelin in school sport tournament.

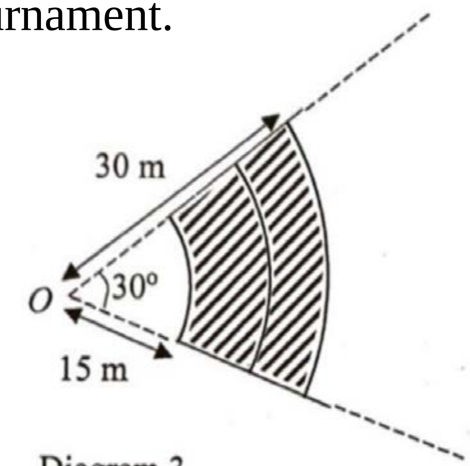


Diagram 3

The distance of the throw is measured from point *O*.

(a) The shaded area is the area where the grass need to be replanted. Calculate the cost needed by the school to replant the grass, if the cost 1m^2 is RM12.50. [4 *marks*]

(b) In the event, Syafiq make a new record with a throw of 44 m. The school management will build an arc to mark the distance that has been achieved by Syafiq. Find the length, in m, of the arc. [2 *marks*]

Solution

(a)

$$30° = 30° \times \frac{3.142}{180°}$$

$$\theta = 0.5237 \text{ rad} \quad //0.5236 \text{ rad}$$

Area shaded region = Big Area Sector − Small Area Sector

$$A = \frac{1}{2}r^2\theta$$

$$A = \frac{1}{2}(30)^2(0.5237) - \frac{1}{2}(15)^2(0.5237) \ ✓ \ ✓$$

$$= 176.75 \text{ cm}^2 \ ✓ \quad 176.7 \leftrightarrow 177$$

$$\text{Cost} = \text{RM}12.50 \times 176.75$$

$$= \text{RM}2209.38 \ ✓ \quad 2208.8 \leftrightarrow 2212.5$$

(b)

$$s = r\theta$$

$$s = 44(0.5237) \ ✓$$

$$= 23.04 \text{ m} \ ✓ \quad 23.03 \leftrightarrow 23.05$$

Example 17

SPMRSM 2020, Paper 2

Diagram 6 shows a semicircle *PQS* with the centre *O* and a sector *PRS* with the centre *P*.

Given that the length of arc *RS* is 14.66 cm and *OP = PQ*, find
[Use $\pi = 3.142$]

(a) the radius, in cm, of semicircle *PQS*, [3 *marks*]
(b) the perimeter, in cm, of the shaded region, [2 *marks*]
(c) the area, in cm^2, of the shaded region. [5 *marks*]

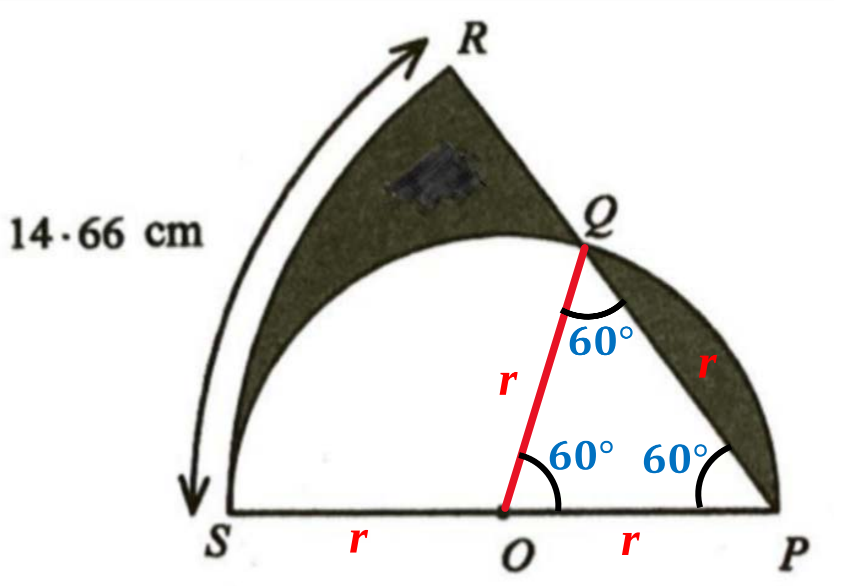
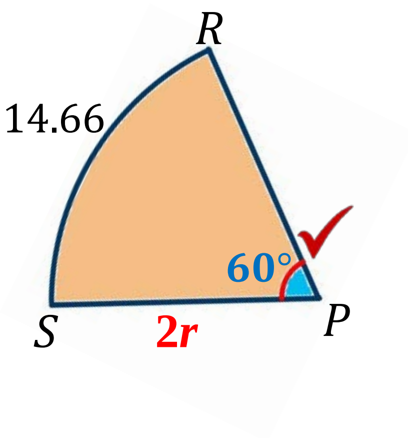


Diagram 6

Solution

θ must be in radian

(a)



$$s = r\theta$$

$$14.66 = PS \times \left(60° \times \frac{3 \cdot 142}{180°}\right)$$

$$14.66 = PS \times 1.047$$

$$PS = 14$$

$$2r = 14$$

$$r = 7 \text{ cm}$$

(b)

Perimeter shaded region = Arc length *PQS* + Arc length *RS* + *PR*

$$P = \pi r + 14.66 + 14$$

$$P = 3.142(7) + 14.66 + 14$$

$$P = 50.65 \text{ cm}$$

Example 17

SPMRSM 2020, Paper 2

Diagram 6 shows a semicircle *PQS* with the centre *O* and a sector *PRS* with the centre *P*.

Given that the length of arc *RS* is 14.66 cm and *OP* = *PQ*, find
[Use $\pi = 3.142$]

(a) the radius, in cm, of semicircle *PQS*, [3 *marks*]
(b) the perimeter, in cm, of the shaded region, [2 *marks*]
(c) the area, in cm^2, of the shaded region. [5 *marks*]

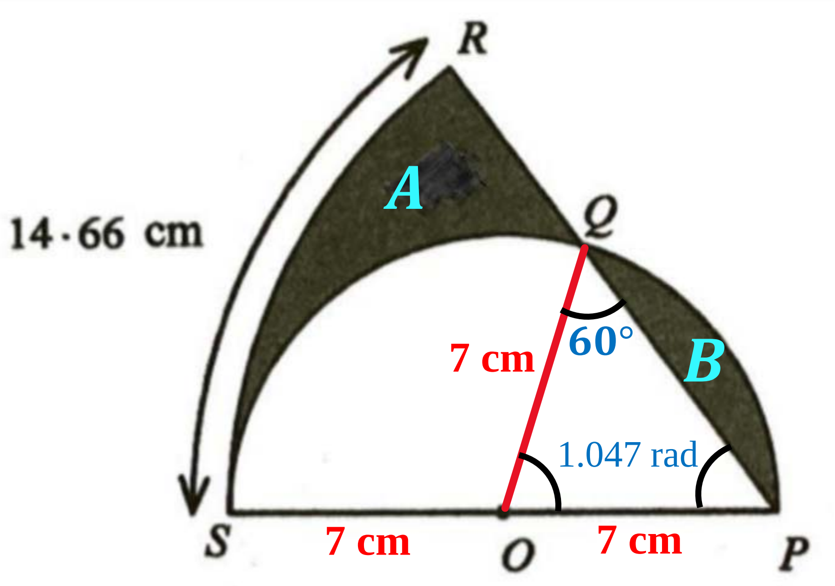


Diagram 6

Solution

(c)

Area shaded region = Area A + Area B

$A = \frac{1}{2}r^2\theta$

$A = \frac{1}{2}ab\sin C$

Area A = Area sector PRS − Area sector QOS − Area ΔPOQ

$= \frac{1}{2}(14)^2(1.047) - \frac{1}{2}(7)^2(3.142 - 1.047) - \frac{1}{2}(7)^2(\sin 60°)$ ✓ ✓

$= 30.0609 \text{ cm}^2$

Area B = Area sector POQ − Area ΔPOQ

$= \frac{1}{2}(7)^2(1.047) - \frac{1}{2}(7)^2(\sin 60°)$ ✓

$= 4.4339 \text{ cm}^2$

Area shaded region = 30.0609 + 4.4339 ✓

$= 34.49 \text{ cm}^2$ ✓

$34.49 \leftrightarrow 34.54$

Example 18

Diagram 5(a) shows a side plan of a mini house model. Tamrin needs to colour the triangular area on the side of the house using special paint, as shown in Diagram 5(b).

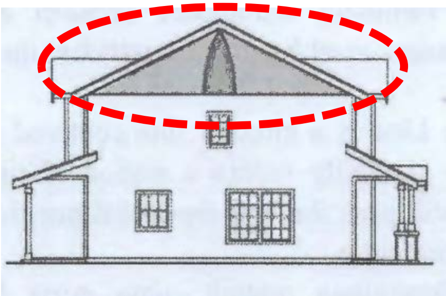


Diagram 5(a)

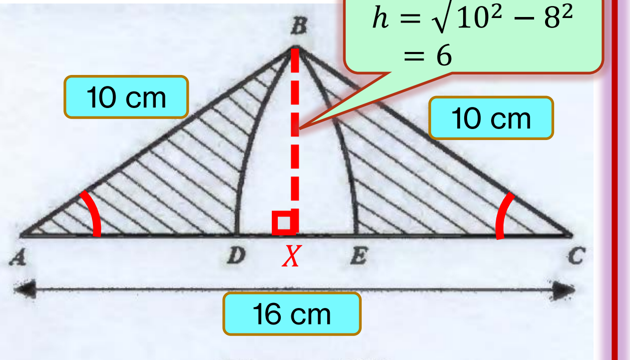


Diagram 5(b)

ABC is an isosceles triangle with $AC = 16$ cm and $AB = BC = 10$ cm. BE and BD are arcs of two circles, with centre A and C respectively. Points D and E lie on straight line AC. Calculate

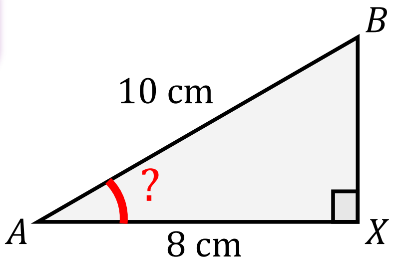
(a) $\angle BAC$ in radians, [2 marks]

(b) the perimeter, in cm, of the shaded region, [3 marks]

(c) the cost needed to paint the region BDE with acrylic paint if the price of the paint is RM 0.35/cm^2. [5 marks]

Solution

SBP 2020, Paper 2



(a)

$\cos \angle BAC = \frac{8}{10}$

$\cos^{-1}\left(\frac{8}{10}\right)$

$\angle BAC = 36.87°$

$= 36.87° \times \frac{\pi}{180°}$

$= 0.6435$ rad // 0.6436 rad

(b)

Perimeter shaded region $= 2(AB + \text{Arc } BD + AD)$

Arc BD = Arc $BE = 10 \times 0.6435$ $\quad s = r\theta$

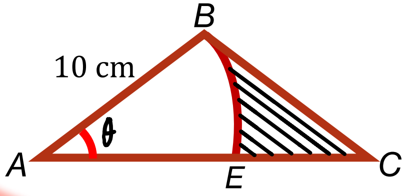
$= 6.435$ cm

$AD = EC = 16 - 10$

$= 6$ cm

Perimeter shaded region $= 2(10 + 6.435 + 6)$

$= 44.87$ cm



(c) Area region BCE = Area ΔABC − Area sector ABE

$= \frac{1}{2} \times 16 \times 6 - \frac{1}{2}(10)^2(0.6435)$ $\quad A = \frac{1}{2}r^2\theta$

$= 15.825$ cm^2

Alternative Method

$A = \frac{1}{2}(10)(16)(\sin 36.87°)$

Area region BDE = Area ΔABC − 2(Area region BCE)

$= \frac{1}{2} \times 16 \times 6 - 2(15.825)$

$= 16.35$ cm^2 // 16.36

Cost = RM 0.35 × 16.35

= RM5.72 // RM5.73

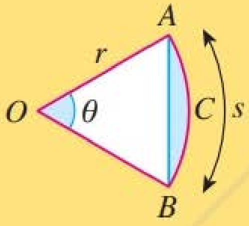
REFLECTION CORNER

CIRCULAR MEASURE

Convert radians into degrees and vice versa

$$\text{Radians} \xrightarrow{\times \frac{180^\circ}{\pi}} \text{Degrees}, \qquad \text{Degrees} \xrightarrow{\times \frac{\pi}{180^\circ}} \text{Radians}$$

Arc length of a circle

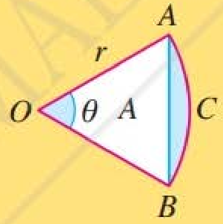


Arc length, $s = r\theta$

Perimeter of segment ABC

$= s + AB$

Area of a sector of a circle



Area of sector, $A = \frac{1}{2}r^2\theta$

Area of segment ABC

$= A$ – area of ΔAOB

Applications

What will ask in the questions?

- angle
- radius
- arc length
- perimeter
- area sector / shaded region

Involves triangle:

length
- cosine rule: $a^2 = b^2 + c^2 - 2bc\cos A$
- trigo ratio
- Pythagoras theorem

Area
- $A = \frac{1}{2}ab\sin C$ **or** $\frac{1}{2}r^2\sin\theta$
- $A = \frac{1}{2} \times b \times h$ (trigo ratio/PT)

PAPER 1:

Section A : 4 – 7 marks

Section B : 8 marks

PAPER 2:

Section A : 6 – 8 marks

Section B : 10 marks